Intuitionistic Fuzzy $\alpha g^{**}$-Closed sets, Intuitionistic Fuzzy $\alpha g^{**}$-Continuity and Intuitionistic Fuzzy $\alpha g^{**}$- Homeomorphisms

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Abstract:
In this paper, we introduce and study the notions of intuitionistic fuzzy $\alpha g^{**}$-closed sets, intuitionistic fuzzy $\alpha g^{**}$-continuity, intuitionistic fuzzy $\alpha g^{**}$-open mapping, intuitionistic fuzzy $\alpha g^{**}$-closed mapping, intuitionistic fuzzy $\alpha g^{**}$- homeomorphisms and some of its properties in intuitionistic fuzzy topological spaces.

Key words: Intuitionistic fuzzy $g^{*}$-closed set, intuitionistic fuzzy $g^{**}$-closed sets, intuitionistic fuzzy $g^{**}$-closed sets, intuitionistic fuzzy $\alpha g^{**}$-continuity, intuitionistic fuzzy $\alpha g^{**}$-open mapping, intuitionistic fuzzy $\alpha g^{**}$-closed mapping and intuitionistic fuzzy $\alpha g^{**}$-homeomorphisms.

1. Introduction
Zadeh [12] introduced the notion of fuzzy sets. Later on, fuzzy topology was introduced by Chang [2] in 1967. The concept of intuitionistic fuzzy topology was introduced by Atanassov [1] as a generalization of fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological space. In this paper, we introduce the concepts of intuitionistic fuzzy $\alpha g^{**}$-closed sets, intuitionistic fuzzy $\alpha g^{**}$-continuity, intuitionistic fuzzy $\alpha g^{**}$-open mapping, intuitionistic fuzzy $\alpha g^{**}$-closed mapping, intuitionistic fuzzy $\alpha g^{**}$-homeomorphisms and study some of its properties in intuitionistic fuzzy topological spaces.

2. Preliminaries
2.1 Definition [1]
An intuitionistic fuzzy set (IFS in short) $A$ in $X$ is an object having the form $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$, where the functions $\mu_A: X \to [0, 1]$ and $\gamma_A: X \to [0, 1]$ denote the the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set $A$ respectively and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$. Denote by IFS($X$), the set of all intuitionistic fuzzy sets in $X$.

2.2 Definition [1]
Let $A$ and $B$ be intuitionistic fuzzy sets of the form $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ and $B = \{< x, \mu_B(x), \gamma_B(x) >: x \in X\}$. Then
1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$. 
2.\text{\ } A = B \text{ if and only if } A \subseteq \text{ and } B \subseteq A.
3.\text{\ } A^c = \{ < x, \gamma_A(x), \mu_A(x) > \mid x \in X \}.
4.\text{\ } A \cap B = \{ < x, \mu_A(x) \land \mu_B(x), \gamma_A(x) \lor \gamma_B(x) > \mid x \in X \}.
5.\text{\ } A \cup B = \{ < x, \mu_A(x) \lor \mu_B(x), \gamma_A(x) \land \gamma_B(x) > \mid x \in X \}.

For the sake of simplicity, we shall use the notation \( A = \{ < x, \mu_A, \gamma_A > \} \) instead of \( A = \{ < x, \mu_A(x), \gamma_A(x) > : x \in X \} \). Also for the sake of simplicity, we shall use the notation \( A = < x, (\mu_A, \mu_B), (\gamma_A, \gamma_B) > \) instead of \( A = < x, (\mu_A / \mu_B, (A / \gamma_A, B / \gamma_B) > \).

The intuitionistic fuzzy sets \( 0_\text{c} = \{ < x, 0, 1 > : x \in X \} \) and \( 1_\text{c} = \{ < x, 1, 0 > : x \in X \} \) are respectively the empty and whole set of \( X \).

### 2.3 Definition [3]

An intuitionistic fuzzy topology (IFT in short) on \( X \) is a family \( \tau \) of IFSs in \( X \) satisfying the following axioms:

1. \( 0_\text{c}, 1_\text{c} \in \tau \),
2. \( G_1 \cap G_2 \in \tau \), for any \( G_1, G_2 \in \tau \),
3. \( \cup G_i \in \tau \), for any family \( \{ G_i / i \in J \} \subseteq \tau \).

In this case the pair \( (X, \tau) \) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS in short) in \( X \). The complement \( A^c \) of an IFOS \( A \) in an IFTS \( (X, \tau) \) is called an intuitionistic closed set (IFCS in short) in \( X \).

### 2.4 Definition [3]

Let \( (X, \tau) \) be an IFTS and \( A = < x, \mu_A, \gamma_A > \) be an IFS in \( X \). Then

1. \( \text{int}(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \} \).
2. \( \text{cl}(A) = \bigcap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \} \).
3. \( \text{cl}(A^c) = (\text{int}(A))^c \).
4. \( \text{int}(A^c) = (\text{cl}(A))^c \).

### 2.5 Result [9]

Let \( A \) be an IFS in \( (X, \tau) \). Then

1. \( \text{acl}(A) = A \cup \text{cl}(\text{int}(cl(A))) \)
2. \( \text{aint}(A) = A \cap \text{int}(\text{cl}(\text{int}(A))) \)

### 2.6 Definition [4]

An IFS \( A = \{ < x, \mu_A(x), \gamma_A(x) > : x \in X \} \) in an IFTS \( (X, \tau) \) is said to be an

1. intuitionistic fuzzy regular open set (IFROS) if \( A = \text{int}(\text{cl}(A)) \).
2. intuitionistic fuzzy \( \alpha \)- open set (IF\( \alpha \text{OS}) if \( A \subseteq \text{int}(\text{cl}(\text{int}(A))) \).

An IFS \( A \) is said to be an intuitionistic fuzzy regular closed set (IFRCS) and intuitionistic fuzzy \( \alpha \)- closed set (IF\( \alpha \text{CS}) if the complement of \( A \) is an IFROS and IF\( \alpha \text{OS} \) respectively.

### 2.7 Definition

An IFS \( A = \{ < x, \mu_A(x), \gamma_A(x) > : x \in X \} \) in an IFTS \( (X, \tau) \) is said to be an

1. intuitionistic fuzzy \( \alpha \)- general closed set (IF\( \alpha \text{GCS}) [6] if \( \text{acl}(A) \subseteq U \text{ whenever } A \subseteq U \) and \( U \) is an IFOS in \( X \).
2. intuitionistic fuzzy regular \( \alpha \)- closed set (IF\( \alpha \text{RGCS}) [10] if \( \text{cl}(A) \subseteq U \text{ whenever } A \subseteq U \) and \( U \) is an IFROS in \( X \).

An IFS \( A \) is said to be an intuitionistic fuzzy \( \alpha \)- general open set (IF\( \alpha \text{GOS}) and intuitionistic fuzzy regular \( \alpha \)- general open set (IF\( \alpha \text{RGOS}) if the complement of \( A \) is a IF\( \alpha \text{GCS} \) and IF\( \alpha \text{RGCS} \) respectively.
2.8 Definition [4]  
Let \((X, \tau)\) and \((Y, \sigma)\) be two intuitionistic fuzzy topological spaces and let \(f : X \to Y\) be a function. Then \(f\) is said to be an intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of \(Y\) is an intuitionistic fuzzy open set in \(X\).

2.9 Definition [9]  
Let \(f\) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \(f\) is said to be an intuitionistic fuzzy \(\alpha\)-continuous if \(f^{-1}(B) \in \text{IF}\alpha\text{O}(X)\) for every \(B \in \sigma\).

2.10 Definition [7]  
Let \((X, \tau)\) and \((Y, \sigma)\) be two intuitionistic fuzzy topological spaces and let \(f : X \to Y\) be a function. Then \(f\) is said to be an intuitionistic fuzzy \(\alpha g\)-continuous if pre image of every intuitionistic fuzzy closed set in \(Y\) is intuitionistic fuzzy \(\alpha g\)-closed in \(X\).

2.11 Definition [11]  
Let \(\alpha, \beta \in [0,1]\) with \(\alpha + \beta \leq 1\). An intuitionistic fuzzy point (briefly IFP), written as \(p_{(\alpha, \beta)}\), is defined to be an IFS of \(X\) given by
\[
p_{(\alpha, \beta)}(x) = \begin{cases} 
(\alpha, \beta), & \text{if } x = \text{I} \\
(0,1), & \text{otherwise}.
\end{cases}
\]
We observe that an IFP \(p_{(\alpha, \beta)}\) is said to belong to an IFS \(A = <x, \mu_A(x), \gamma_A(x)>\), denoted by \(p_{(\alpha, \beta)} \in A\) if \(\alpha \leq \mu_A(x)\) and \(\beta \geq \gamma_A(x)\).

2.12 Definition [11]  
Two IFSs \(A\) and \(B\) are said to be \(q\)-coincident (\(A_q B\) in short) if and only if there exists an element \(x \in X\) such that \(\mu_A(x) > \gamma_B(x)\) or \(\gamma_A(x) < \mu_B(x)\).

2.13 Definition [11]  
Two IFSs are said to be not \(q\)-coincident (\(A_q^c B\) in short) if and only if \(A \not\subseteq B^c\).

2.14 Definition [4]  
Let \((X, \tau)\) and \((Y, \sigma)\) be two intuitionistic fuzzy topological spaces and let \(f : X \to Y\) be a function. Then \(f\) is said to be an
\begin{itemize}
  
  \item[(i)] Intuitionistic fuzzy closed map if the image of each intuitionistic fuzzy closed set in \(X\) is an intuitionistic fuzzy closed set in \(Y\).
  
  \item[(ii)] Intuitionistic fuzzy open map if the image of each intuitionistic fuzzy open set in \(X\) is an intuitionistic fuzzy open set in \(Y\).
\end{itemize}

2.15 Definition [5]  
Let \(f\) be a mapping from an IFTS \((X, \tau)\) into an IFTS\((Y, \sigma)\). Then \(f\) is said to be an intuitionistic fuzzy \(\alpha\)-closed mapping (IF\(\alpha\)-closed mapping in short) if \(f(A)\) is an IF\(\alpha\)CS in \(Y\) for every IFCS \(A\) in \(X\).

2.16 Definition [8]  
Let \(f\) be a bijection mapping from an IFTS \((X, \tau)\) into an IFTS\((Y, \sigma)\). Then \(f\) is said to be an
\begin{itemize}
  
  \item[(i)] Intuitionistic fuzzy homeomorphism (IF homeomorphism in short) if \(f\) and \(f^{-1}\) are IF continuous mappings.
  
  \item[(ii)] Intuitionistic fuzzy \(\alpha\)-homeomorphism (IF\(\alpha\)-homeomorphism in short) if \(f\) and \(f^{-1}\) are IF\(\alpha\)- continuous mappings.
\end{itemize}
3. Intuitionistic Fuzzy $ag^{**}$-Closed sets

In this section, we introduced the concept of intuitionistic fuzzy $ag^{**}$-closed sets and studied some of its properties in intuitionistic fuzzy topological spaces.

3.1 Definition An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $g^{*}$-closed set (briefly IF$g^{*}$CS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFGOS in $X$.

3.2 Example Let $X = \{a, b\}$ and $\tau = \{0_-, A, 1_\}$ be an IFTS on $X$, where $A = \langle x, (0.5, 0.6), (0.3, 0.2) \rangle$. Then $S = \langle x, (0.2, 0.2), (0.6, 0.6) \rangle$ is an IF$g^{*}$CS in $X$.

3.3 Definition An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $g^{**}$-open set (briefly IF$g^{**}$OS) if the complement is an IF$g^{*}$CS in $X$.

3.4 Example Let $X = \{a, b\}$ and $\tau = \{0_-, A, 1_\}$ be an IFTS on $X$, where $A = \langle x, (0.5, 0.6), (0.3, 0.2) \rangle$. Then $S = \langle x, (0.2, 0.2), (0.6, 0.7) \rangle$ is an IF$g^{**}$OS in $X$.

3.5 Definition An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $g^{**}$-closed set (briefly IF$g^{**}$CS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IF$g^{**}$OS in $X$.

3.6 Example Let $X = \{a, b\}$ and $\tau = \{0_-, A, 1_\}$ be an IFTS on $X$, where $A = \langle x, (0.5, 0.6), (0.3, 0.2) \rangle$. Then $S = \langle x, (0.2, 0.2), (0.6, 0.7) \rangle$ is an IF$g^{**}$CS in $X$.

3.7 Definition An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $g^{**}$-open set (briefly IF$g^{**}$OS) if the complement is an IF$g^{**}$CS in $X$.

3.8 Example Let $X = \{a, b\}$ and $\tau = \{0_-, A, 1_\}$ be an IFTS on $X$, where $A = \langle x, (0.5, 0.6), (0.3, 0.2) \rangle$. Then $S = \langle x, (0.2, 0.2), (0.6, 0.7) \rangle$ is an IF$g^{**}$CS in $X$.

3.9 Theorem Every IFCS (resp. IFOS) is an IF$g^{**}$CS (resp. IF$g^{**}$OS) but not conversely.

Proof: Let $A \subseteq U$ and $U$ is IFOS in $(X, \tau)$. Since $\alpha cl(A) \subseteq cl(A)$ and $A$ is an IFCS, $\alpha cl(A) \subseteq cl(A) = A \subseteq U$. Therefore $A$ is an IF $g^{**}$CS in $X$.

3.10 Example (i) Let $X = \{a, b\}$ and $\tau = \{0_-, A, 1_\}$ be an IFTS on $X$, where $A = \langle x, (0.2, 0.4), (0.1, 0.2) \rangle$. Then $S = \langle x, (0.1, 0.2), (0.4, 0.5) \rangle$ is an IF$g^{**}$CS but not IFCS in $X$.

3.11 Definition An IFS $A$ of an IFTS $(X, \tau)$ is said to be intuitionistic fuzzy $ag^{**}$-closed set (briefly IF$ag^{**}$CS) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is IF$g^{**}$OS in $X$. The set of all IF$ag^{**}$CSs of $X$ is denoted by IF$ag^{**}$C$(X)$.

3.12 Example Let $X = \{a, b\}$ and $\tau = \{0_-, A, 1_\}$ be an IFTS on $X$, where $A = \langle x, (0.5, 0.6), (0.3, 0.2) \rangle$. Then the IFS $S = \langle x, (0.3, 0.2), (0.5, 0.6) \rangle$ is an IF$ag^{**}$CS in $(X, \tau)$.

3.13 Theorem Every IFCS, IFRCS and IF$\alpha$CS in $(X, \tau)$ is an IF$ag^{**}$CS, but not conversely.

Proof: Obvious

3.14 Example (i) Let $X = \{a, b\}$ and $\tau = \{0_-, A, 1_\}$ be an IFTS on $X$ where $A = \langle x, (0.5, 0.6), (0.3, 0.2) \rangle$. Then $S = \langle x, (0.2, 0.2), (0.5, 0.6) \rangle$ is an IF$ag^{**}$CS but not an IFCS and IFRCS in $X$. 

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(ii) Let $X = \{a, b\}$ and $\tau = \{0,_, A, 1,_,\}$ be an IFTS on $X$ where $A = < x, (0.2, 0.1), (0.2, 0.4) >$. Then $S = < x, (0.2, 0.2), (0.6, 0.8) >$ is an IF$\alpha g^{*}$CS but not an IF$\alpha$CS in $X$.

3.15 Theorem Every IF$\alpha g^{*}$CS in $(X, \tau)$ is an IF$\alpha$GCS, but not conversely.

Proof: Let $A \subseteq U$ where $U$ is IFOS in $X$. Since every IFOS is an IF$g^{*}$OS and $A$ is IF$\alpha g^{*}$CS we have $acl(A) \subseteq U$. Hence $A$ is an IF$\alpha$GCS in $X$.

3.16 Example Let $X = \{a, b\}$ and $\tau = \{0,_, A, 1,_,\}$ be an IFTS on $X$, where $A = < x, (0.5, 0.4), (0.2, 0.1) >$. Then the IFS $S = < x, (0.6, 0.3), (0.2, 0.1) >$ is an IF$\alpha$GCS in $(X, \tau)$ but not an IF$\alpha g^{*}$CS in $X$.

3.17 Remark
From the above theorems, we have the following diagram.

```
IFCS
<p>| |</p>
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IF$\alpha$CS
```

where $A \rightarrow B$ represents $A$ imply $B$, $A \nRightarrow B$ represents $A$ does not imply $B$.

3.18 Theorem Union of two IF$\alpha g^{*}$CSs again an IF$\alpha g^{*}$CS.

Proof: Let $U$ be an IF$g^{*}$OS in $X$, such that $A \cup B \subseteq U$. Since $A$ and $B$ are IF$\alpha g^{*}$CSs we have $acl(A) \subseteq U$ and $acl(B) \subseteq U$. Therefore $acl(A) \cup acl(B) \subseteq acl(A \cup B) \subseteq U$. Hence $A \cup B$ is an IF$\alpha g^{*}$CS in $(X, \tau)$.

3.19 Theorem If $A$ is an IF$\alpha g^{*}$CS and $A \subseteq B \subseteq acl(A)$ then $B$ is an IF$\alpha g^{*}$CS.

Proof: Let $U$ be an IF$g^{*}$OS such that $\subseteq U$ . Since $A$ is an IF$\alpha g^{*}$CS, we have $acl(A) \subseteq U$. By hypothesis $B \subseteq acl(A)$ then $acl(B) \subseteq acl(A)$. This implies $acl(B) \subseteq U$. Hence $B$ is an IF$\alpha g^{*}$CS.

3.20 Definition An IFS $A$ of an IFTS $(X, \tau)$ is called an intuitionistic fuzzy $g^{*}$ open set (IF$g^{*}$OS in short) if and only if $A^{c}$ is an IF$\alpha g^{*}$CS in $(X, \tau)$.

3.21 Theorem For any IFTS $(X, \tau)$, we have the following:

(i) Every IFOS is an IF$\alpha g^{*}$OS
(ii) Every IF$\alpha$OS is an IF$\alpha g^{*}$OS
(iii) Every IFROS is an IF$\alpha g^{*}$OS

Proof: Obvious.

3.21 Theorem If $A$ is an IF$g^{*}$OS and $aint(A) \subseteq B \subseteq A$, then $B$ is an IF$\alpha g^{*}$OS.

Proof: If $aint(A) \subseteq B \subseteq A$, then $A^{c} \subseteq B^{c} \subseteq (aint(A))^{c} = acl(A^{c})$. Since $A^{c}$ is an IF$\alpha g^{*}$CS, then by Theorem 3.21, $B^{c}$ is an IF$\alpha g^{*}$CS. Therefore $B$ is an IF$\alpha g^{*}$OS.

4. Intuitionistic Fuzzy $\alpha^{*}$g- Continuity
In this section we introduced the concept of intuitionistic fuzzy $\alpha^{*}$g -continuous mapping and studied some of its properties.
4.1 Definition A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an intuitionistic fuzzy \( \alpha g^{**} \)-continuous (briefly IF\( \alpha g^{**} \)-continuous) if inverse image of every intuitionistic fuzzy closed set of \( Y \) is an intuitionistic fuzzy \( \alpha g^{**} \)-closed set in \( X \).

4.2 Example Let \( X= \{ a, b \} \), \( Y=\{ u, v \} \) and \( A= < x, (0.1, 0.2), (0.2, 0.2) > \), \( B= < y, (0.5,0.6), (0.1,0.1) > \). Then \( \tau = \{ 0_-, A, 1_- \} \), \( \sigma = \{ 0_-, B, 1_- \} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF\( \alpha g^{**} \)-continuous mapping.

4.3 Theorem A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IF\( \alpha g^{**} \)-continuous if and only if the inverse image of every IFOS of \( Y \) is an IFOS in \( X \).

**Proof:** It is obvious, because \( f^{-1}(B^c) = (f^{-1}(B))^c \) for every IFS \( B \) of \( Y \).

4.4 Theorem Every intuitionistic fuzzy continuous mapping is IF\( \alpha g^{**} \)-continuous, but converse may not be true.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an intuitionistic fuzzy continuous mapping. Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is IFCS. Since every IFCS is IF\( g^{**} \)CS, \( f^{-1}(A) \) is an IF\( g^{**} \)CS in \( X \). Hence \( f \) is an IF\( \alpha g^{**} \)-continuous mapping.

4.5 Example Let \( X= \{ a, b \} \), \( Y=\{ u, v \} \) and \( A= < x, (0.5, 0.6), (0.3, 0.2) > \), \( B= < y, (0.5,0.6), (0.2,0.2) > \). Then \( \tau = \{ 0_-, A, 1_- \} \), \( \sigma = \{ 0_-, B, 1_- \} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The intuitionistic fuzzy set \( S = < y, (0.2, 0.1), (0.2, 0.4) > \) is IFCS in \( Y \). Then \( f^{-1}(S) \) is IF\( g^{**} \)CS in \( X \) but not IFCS in \( X \). Therefore, \( f \) is an IF\( \alpha g^{**} \)-continuous mapping but not an intuitionistic fuzzy continuous mapping.

4.6 Theorem Every intuitionistic fuzzy \( \alpha \)-continuous mapping is IF\( \alpha g^{**} \)-continuous, but converse may not be true.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an intuitionistic fuzzy \( \alpha \)-continuous mapping. Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is IF\( \alpha \)CS. Since every IF\( \alpha \)CS is IF\( g^{**} \)CS, \( f^{-1}(A) \) is an IF\( g^{**} \)CS in \( X \). Hence \( f \) is an IF\( \alpha g^{**} \)-continuous mapping.

4.7 Example Let \( X= \{ a, b \} \), \( Y=\{ u, v \} \) and \( A= < x, (0.2, 0.1), (0.2, 0.4) > \), \( B= < y, (0.2,0.4), (0.2,0.1) > \). Then \( \tau = \{ 0_-, A, 1_- \} \), \( \sigma = \{ 0_-, B, 1_- \} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The intuitionistic fuzzy set \( S = < y, (0.2, 0.1), (0.2, 0.4) > \) is IFCS in \( Y \). Then \( f^{-1}(S) \) is IF\( g^{**} \)CS in \( X \) but not IF\( \alpha \)CS in \( X \). Therefore, \( f \) is an IF\( \alpha g^{**} \)-continuous mapping but not an intuitionistic fuzzy \( \alpha \)-continuous mapping.

4.8 Theorem Every intuitionistic fuzzy \( \alpha g^{**} \)-continuous mapping is IF\( \alpha G \)-continuous, but converse may not be true.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an intuitionistic fuzzy \( \alpha g^{**} \)-continuous mapping. Let \( A \) be an IFCS in \( Y \). By hypothesis, \( f^{-1}(A) \) is an IF\( \alpha G \)CS in \( X \). Hence \( f \) is an IF\( \alpha g^{**} \)-continuous mapping.

4.9 Example Let \( X= \{ a, b \} \), \( Y=\{ u, v \} \) and \( A= < x, (0.5, 0.4), (0.2, 0.1) > \), \( B= < y, (0.2,0.1), (0.6,0.3) > \). Then \( \tau = \{ 0_-, A, 1_- \} \), \( \sigma = \{ 0_-, B, 1_- \} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The intuitionistic fuzzy set \( S = < y, (0.6, 0.3), (0.2, 0.1) > \) is IFCS in \( Y \).
Then \( f^{-1}(S) \) is IF\( \alpha \)GCS in \( X \) but not IF\( \alpha \)g**CS in \( X \). Therefore \( f \) is an IF\( \alpha \)G -continuous mapping but not an intuitionistic fuzzy \( \alpha \)g**-continuous mapping.

The relation between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram ‘cts.’ means continuous.

\[
\text{IF-cts.} \quad \text{IF}\alpha\text{-cts} \quad \text{IF}\alpha\text{G-cts.} \quad \text{IF}\alpha\text{g**-cts.}
\]

4.10 Theorem If \( f: (X, \tau) \to (Y, \sigma) \) is an IF\( \alpha \)g** -continuous then for each IFP \( p_{(\alpha, \beta)} \) of \( X \) and each IFOS \( B \) of \( Y \) such that \( f(p_{(\alpha, \beta)}) \subseteq B \) there exists an intuitionistic fuzzy \( \alpha \)g**-open set \( A \) of \( X \) such that \( p_{(\alpha, \beta)} \subseteq A \) and \( f(A) \subseteq B \).

**Proof:** Let \( p_{(\alpha, \beta)} \) be an IFP of \( X \) and \( B \) be an IFOS of \( Y \) such that \( f(p_{(\alpha, \beta)}) \subseteq B \). Put \( A = f^{-1}(B) \). Then by hypothesis \( A \) is an intuitionistic fuzzy \( \alpha \)g**-open set of \( X \) such that \( p_{(\alpha, \beta)} \subseteq A \) and \( f(A) = f(f^{-1}(B)) \subseteq B \).

4.11 Theorem If \( f: (X, \tau) \to (Y, \sigma) \) is an IF\( \alpha \)g** -continuous then for each IFP \( p_{(\alpha, \beta)} \) of \( X \) and each IFOS \( B \) of \( Y \) such that \( f(p_{(\alpha, \beta)})_q \subseteq B \) there exists an intuitionistic fuzzy \( \alpha \)g**-open set \( A \) of \( X \) such that \( p_{(\alpha, \beta)}_q A \) and \( f(A) = f(f^{-1}(B)) \subseteq B \).

4.12 Definition Let \( (X, \tau) \) be an IFTS and \( A \) be an IFS in \( X \). Then \( \alpha \)g**- interior and \( \alpha \)g**-closure of \( A \) are defined as

\[
\alpha g^{**} \text{int}(A) = \cap \{ K: K \text{ is an IF}\alpha g^{**}\text{CS in } X \text{ and } A \subseteq K \}
\]

\[
\alpha g^{**} \text{cl}(A) = \cup \{ G: G \text{ is an IF}\alpha g^{**}\text{OS in } X \text{ and } G \subseteq A \}
\]

If \( A \) is IF\( \alpha \)g**CS, then \( \alpha g^{**} \text{int} (A) = A \).

4.13 Theorem If \( f: (X, \tau) \to (Y, \sigma) \) is IF \( \alpha \)g** - continuous and \( g: (Y, \sigma) \to (Z, \mu) \) in intuitionistic fuzzy continuous, then \( g \circ f: (X, \tau) \to (Z, \mu) \) is IF\( \alpha \)g** - continuous.

**Proof:** Let \( A \) be an IFCS in \( Z \). Then \( g^{-1}(A) \) is an IFCS in \( Y \) because \( g \) is intuitionistic fuzzy continuous. Therefore, \( (g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A)) \) is an IF\( \alpha \)g**CS in \( X \). Hence \( g \circ f \) is IF\( \alpha \)g** - continuous.

4.14 Theorem Let \( f: (X, \tau) \to (Y, \sigma) \) be a mapping and let \( f^{-1}(A) \) be an IFRCS in \( X \) for every IFCS \( A \) in \( Y \). Then \( f \) is an IF\( \alpha \)g** -continuous mapping.

**Proof:** Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFRCS in \( X \). Since every IFRCS is an IF\( \alpha \)g**CS, \( f^{-1}(A) \) is an IF\( \alpha \)g**CS in \( X \). Hence \( f \) is an IF\( \alpha \)g** -continuous mapping.

4.15 Theorem Let \( f: (X, \tau) \to (Y, \sigma) \) be an IF\( \alpha \)g** - continuous mapping. Then the following conditions are hold:

(i) \( f(\alpha g^{**}\text{int}(A)) \subseteq \text{cl}(f(A)) \), for every IFS \( A \) in \( X \)

(ii) \( \alpha g^{**}\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B)) \), for every IFS \( B \) in \( Y \).

**Proof:** (i) Since \( \text{cl}(f(A)) \) is an IFCS in \( Y \) and \( f \) is an IF\( \alpha \)g** -continuous mapping, then \( f^{-1}(\text{cl}(f(A))) \) is IF\( \alpha \)g**CS in \( X \). That is \( \alpha g^{**}\text{cl}(A) \subseteq f^{-1}(\text{cl}(f(A))) \). Therefore
5. Intuitionistic Fuzzy ag**- Open Mapping

In this section we introduced the concept of intuitionistic fuzzy ag**- open mapping and studied some of its properties.

5.1 Definition A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an intuitionistic fuzzy ag**-open mapping (briefly IF ag**-open mapping) if the image of every IFOS in X is IF ag** OS in Y.

5.2 Example Let \( X = \{a, b\} \), \( Y = \{x, y\} \) and \( A = \langle x, (0.2, 0.1), (0.6, 0.3) \rangle \), \( B = \langle y, (0.2, 0.1), (0.5, 0.2) \rangle \). Then \( \tau = \{0_-, A, 1_+\} \), \( \sigma = \{0_-, B, 1_+\} \) are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = x \) and \( f(b) = y \). Then \( f \) is an IF ag** -open mapping.

5.3 Theorem Every intuitionistic fuzzy open map is an IF ag** -open map but converse may not be true.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an intuitionistic fuzzy open mapping. Let A be an IFOS in X. Since \( f \) is an intuitionistic fuzzy open mapping, \( f(A) \) is an IFOS in Y. Since every IFOS is an IF ag** OS, \( f(A) \) is an IF ag** OS in Y. Hence \( f \) is an IF ag** -open mapping.

5.4 Example Let \( X = \{a, b\} \), \( Y = \{x, y\} \) and \( A = \langle x, (0.2, 0.1), (0.6, 0.3) \rangle \), \( B = \langle y, (0.2, 0.1), (0.5, 0.2) \rangle \). Then \( \tau = \{0_-, A, 1_+\} \), \( \sigma = \{0_-, B, 1_+\} \) are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = x \) and \( f(b) = y \). Then \( f \) is an IF ag** -open mapping but not an intuitionistic fuzzy open mapping.

5.5 Theorem \( f: (X, \tau) \rightarrow (Y, \sigma) \) is IF ag** -open map then \( \text{int}(f^{-1}(G)) \subseteq f^{-1}(\text{ag**int}(G)) \) for every IF G of Y.

Proof: Let G be an IFOS of Y. Then \( \text{int}(f^{-1}(G)) \) is an IFOS in X. Since \( f \) is an IF ag**-open map \( f(\text{int}(f^{-1}(G))) \) is IF ag** OS in Y and hence \( f(\text{int}(f^{-1}(G))) \subseteq \text{ag**int}(f(f^{-1}(G))) \subseteq \text{ag**int}(G) \). Thus \( \text{int}(f^{-1}(G)) \subseteq f^{-1}(\text{ag**int}(G)) \).

5.6 Theorem A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IF ag** -open if and only if for each IF S of Y and for each IFCS U of X containing \( f^{-1}(S) \) there is an IF ag** CS V of Y such that \( S \subseteq V \) and \( f^{-1}(V) \subseteq U \).

Proof: 

Necessity: Suppose that \( f \) is an IF ag** -open map. Let S be the IFCS of Y and U be an IFCS of X such that \( f^{-1}(S) \subseteq U \). Then \( V = (f^{-1}(U^c))^c \) is an IF ag** CS of Y such that \( f^{-1}(V) \subseteq U \).

Sufficiency: Suppose that F is an IFOS of X. Then \( f^{-1}(F)^c \subseteq F^c \) and \( F^c \) is an IFCS in X. By hypothesis there is an IF ag** CS V of Y such that \( (f(F))^c \subseteq V \) and \( f^{-1}(V) \subseteq F^c \). Therefore \( F \subseteq (f^{-1}(V))^c \). Hence \( V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c \) which implies \( f(F) = V^c \), since \( V^c \) is an IF ag** OS of Y. Hence \( f(F) \) is an IF ag** OS in Y and thus \( f \) is an IF ag** -open map.

5.7 Theorem A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IF ag** -open if and only if \( f^{-1}(\text{ag**cl}(B)) \subseteq \text{cl}(f^{-1}(B)) \) for every IF B of Y.
Proof:

**Necessity:** Suppose that \( f \) is an IF\( \alpha g^{**} \) -open map. For any IFS \( B \) of \( Y \), \( f^{-1}(B) \subseteq cl(f^{-1}(B)) \). Therefore by Theorem (5.6) there exists an IF\( \alpha g^{**} \)CS \( F \) of \( Y \) such that \( B \subseteq F \) and \( f^{-1}(F) \subseteq cl(f^{-1}(B)) \). Therefore we obtain that \( f^{-1}(\alpha g^{**} cl(B)) \subseteq f^{-1}(F) \subseteq cl(f^{-1}(B)) \).

**Sufficiency:** Suppose that \( B \) is an IFS of \( Y \) and \( F \) is an IFCS of \( X \) containing \( f^{-1}(B) \). Put \( V = cl(B) \), then \( B \subseteq V \) and \( V \) is IF\( \alpha g^{**} \)CS and \( f^{-1}(V) \subseteq cl(f^{-1}(B)) \subseteq F \). Then by Theorem (5.6) \( f \) is an IF\( \alpha g^{**} \) -open map.

6. **Intuitionistic Fuzzy \( \alpha g^{**} \) - Closed Mapping**

In this section we introduced the concept of intuitionistic fuzzy \( \alpha g^{**} \) - closed mapping and studied some of its properties.

6.1 **Definition** A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an intuitionistic fuzzy \( \alpha g^{**} \)-closed mapping (briefly IF\( \alpha g^{**} \)-closed mapping) if the image of every IFCS in \( X \) is IF\( \alpha g^{**} \)CS in \( Y \).

6.2 **Example** Let \( X = \{a, b\} \), \( Y = \{x, y\} \) and \( A = < x, (0.5, 0.6), (0.2, 0.1) > \), \( B = < y, (0.5, 0.4), (0.2, 0.1) > \). Then \( \tau = \{0, A, 1\} \), \( \sigma = \{0, B, 1\} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = x \) and \( f(b) = y \). Then \( f \) is an IF\( \alpha g^{**} \) -closed mapping.

**Theorem 6.3:** Every intuitionistic fuzzy closed map is an IF\( \alpha g^{**} \) -closed map but converse may not be true.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an intuitionistic fuzzy closed mapping. Let \( A \) be an IFCS in \( X \). Since \( f \) is an intuitionistic fuzzy closed mapping, \( f(A) \) is an IFCS in \( Y \). Since every IFCS is an IF\( \alpha g^{**} \)CS, \( f(A) \) is an IF\( \alpha g^{**} \)GCS in \( Y \). Hence \( f \) is an IF\( \alpha g^{**} \) -closed mapping.

6.4 **Example** Let \( X = \{a, b\} \), \( Y = \{x, y\} \) and \( A = < x, (0.5, 0.6), (0.2, 0.1) > \), \( B = < y, (0.5, 0.4), (0.2, 0.1) > \). Then \( \tau = \{0, A, 1\} \), \( \sigma = \{0, B, 1\} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = x \) and \( f(b) = y \). Then \( f \) is an IF\( \alpha g^{**} \) -closed mapping but not intuitionistic fuzzy closed mapping.

6.5 **Theorem** Every intuitionistic fuzzy \( \alpha \)-closed map is an IF\( \alpha g^{**} \) -closed map but converse may not be true.

**Proof:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an intuitionistic fuzzy \( \alpha \)-closed mapping. Let \( A \) be an IFCS in \( X \). Since \( f \) is an intuitionistic fuzzy \( \alpha \)-closed mapping, \( f(A) \) is an IF\( \alpha g^{**} \)CS in \( Y \). Since every IF\( \alpha \)CS is an IF\( \alpha g^{**} \)CS, \( f(A) \) is an IF\( \alpha g^{**} \)GCS in \( Y \). Hence \( f \) is an IF\( \alpha g^{**} \) -closed mapping.

6.6 **Example** Let \( X = \{a, b\} \), \( Y = \{x, y\} \) and \( A = < a, (0.5, 0.2), (0.5, 0.8) > \), \( B = < x, (0.2, 0.4), (0.3, 0.6) > \). Then \( \tau = \{0, A, 1\} \), \( \sigma = \{0, B, 1\} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = x \) and \( f(b) = y \). Then \( f \) is an IF\( \alpha g^{**} \)-closed mapping but not intuitionistic fuzzy \( \alpha \)-closed mapping.

6.7 **Theorem** A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is an IF\( \alpha g^{**} \) -closed mapping if and only if the image of each IFOS in \( X \) is an IF\( \alpha g^{**} \)OS in \( Y \).

**Proof:** Let \( A \) be an IFOS in \( X \). This implies \( A^c \) is IFCS in \( X \). Since \( f \) is an IF\( \alpha g^{**} \) -closed mapping, \( f(A^c) \) is an IF\( \alpha g^{**} \)CS in \( Y \). Since \( f(A^c) = (f(A))^c \), \( f(A) \) is an IF\( \alpha g^{**} \)OS in \( Y \).
The relation between various types of intuitionistic fuzzy closed mappings are given in the following diagram:

intuitionistic fuzzy closed mapping \[\rightsquigarrow\] intuitionistic fuzzy \(\alpha\)-closed mapping

\[\rightsquigarrow\] intuitionistic fuzzy \(\alpha g^{**}\)-closed mapping

\(6.8\) Theorem If \(f: (X, \tau) \rightarrow (Y, \sigma)\) is an intuitionistic fuzzy closed map and \(g: (Y, \sigma) \rightarrow (Z, \mu)\) is an IF\(\alpha g^{**}\)-closed map, then \(g \circ f: (X, \tau) \rightarrow (Z, \mu)\) is IF\(\alpha g^{**}\)-closed mapping.

**Proof:** Let \(H\) be an IFCS of an IFTS \((X, \tau)\). Then \(f(H)\) is IFCS of \((Y, \sigma)\), because \(f\) is intuitionistic fuzzy closed map. Now \(g \circ f(H) = g(f(H))\) is an IF\(\alpha g^{**}\)CS in \((Z, \mu)\) because \(g\) is IF\(\alpha g^{**}\)-closed map. Thus \(g \circ f: (X, \tau) \rightarrow (Z, \mu)\) is IF\(\alpha g^{**}\)-closed mapping.

\(6.9\) Theorem Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) and \(g: (Y, \sigma) \rightarrow (Z, \mu)\) are two intuitionistic fuzzy mappings such that their composition \(g \circ f: (X, \tau) \rightarrow (Z, \mu)\) is IF -closed mapping. If \(f\) is intuitionistic fuzzy continuous and surjective, then \(g\) is IF\(\alpha g^{**}\)-closed.

**Proof:** Let \(A\) be an IFCS of \(Y\). Since \(f\) is intuitionistic fuzzy continuous \(f^{-1}(A)\) is IFCS in \(X\). Since \(g \circ f\) is IF\(\alpha g^{**}\)-closed, \(g \circ f(f^{-1}(A))\) is intuitionistic fuzzy \(\alpha g^{**}\)-closed in \(Z\). That is \(g(A)\) is IF\(\alpha g^{**}\)-closed in \(Y\), because \(f\) is surjective. Therefore \(g\) is IF\(\alpha g^{**}\)-closed.

7. Intuitionistic Fuzzy \(\alpha g^{**}\)-Homeomorphisms

In this section we introduced the concept of intuitionistic fuzzy \(\alpha g^{**}\)-homeomorphisms and studied some of its properties.

7.1 Definition A bisection mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called an intuitionistic fuzzy \(\alpha g^{**}\)-homeomorphism (IF\(\alpha g^{**}\)-homeomorphism in short) if \(f\) and \(f^{-1}\) are IF\(\alpha g^{**}\)-continuous mappings.

7.2 Example Let \(X = \{a, b\}\), \(Y = \{x, y\}\) and \(A = \{x, (0.2, 0.6), (0.6, 0.2)\}\), \(B = \{y, (0.3, 0.7), (0.7, 0.3)\}\). Then \(\tau = \{0_-, A, 1_+\}\), \(\sigma = \{0_-, B, 1_+\}\) are intuitionistic fuzzy topologies on \(X\) and \(Y\) respectively. Define a mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) by \(f(a) = x\) and \(f(b) = y\). Then \(f\) is an intuitionistic fuzzy \(\alpha g^{**}\)-homeomorphism.

**Theorem 7.3:** Every IF homeomorphism is an IF\(\alpha g^{**}\)-homeomorphism but converse may not be true.

**Proof:** Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be an IF homeomorphism. Then \(f\) and \(f^{-1}\) are IF continuous mappings. This implies \(f\) and \(f^{-1}\) are IF\(\alpha^{**}\)G-continuous mappings. Hence \(f\) is IF\(\alpha g^{**}\)-homeomorphism.

7.4 Example Let \(X = \{a, b\}\), \(Y = \{x, y\}\) and \(A = \{x, (0.5, 0.6), (0.3, 0.2)\}\), \(B = \{y, (0.5, 0.6), (0.2, 0.2)\}\). Then \(\tau = \{0_-, A, 1_+\}\), \(\sigma = \{0_-, B, 1_+\}\) are intuitionistic fuzzy topologies on \(X\) and \(Y\) respectively. Define a mapping \(f: (X, \tau) \rightarrow (Y, \sigma)\) by \(f(a) = x\) and \(f(b) = y\). Then \(f\) is intuitionistic fuzzy \(\alpha g^{**}\)-homeomorphism but not an IF homeomorphism.

**Theorem 7.5** Every \(\alpha\) homeomorphism is an IF\(\alpha g^{**}\)-homeomorphism but converse may not be true.

**Proof:** Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be an \(\alpha\)-homeomorphism. Then \(f\) and \(f^{-1}\) are \(\alpha\)-continuous mappings. This implies \(f\) and \(f^{-1}\) are IF\(\alpha g^{**}\)-continuous mappings. Hence \(f\) is IF\(\alpha g^{**}\)-homeomorphism.
7.6 Example Let \( X = \{a, b\} \), \( Y = \{x, y\} \) and \( A = \{0.2, 0.4\} \), \( B = \{0.6, 0.8\} \). Then \( \tau = \{0.3, A_{1}\} \), \( \sigma = \{0.4, B_{1}\} \) are intuitionistic fuzzy topologies on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = x \) and \( f(b) = y \). Then \( f \) is an intuitionistic fuzzy \( \alpha \)-homeomorphism.

7.7 Theorem Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a bijective mapping. If \( f \) is an \( \alpha \)-continuous mapping, then the following are equivalent.

(i) \( f \) is an \( \alpha \)-closed mapping

(ii) \( f \) is an \( \alpha \)-open mapping

(iii) \( f \) is an \( \alpha \)-homeomorphism.

Proof: (i)\(\rightarrow\)(ii): Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a bijective mapping and let \( f \) be an \( \alpha \)-closed mapping. This implies \( f^{-1}: (Y, \sigma) \rightarrow (X, \tau) \) is \( \alpha \)-continuous mapping. That is every IFOS in \( X \) is an \( \alpha \)-closed in \( Y \). Hence \( f \) is an \( \alpha \)-open mapping.

(ii)\(\rightarrow\)(iii): Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a bijective mapping and let \( f \) be an \( \alpha \)-open mapping. This implies \( f^{-1}: (Y, \sigma) \rightarrow (X, \tau) \) is \( \alpha \)-continuous mapping. But \( f \) is an \( \alpha \)-continuous mapping by hypothesis. Hence \( f \) and \( f^{-1} \) are \( \alpha \)-continuous mappings. Thus, \( f \) is \( \alpha \)-homeomorphism.

(iii)\(\rightarrow\)(i): Let \( f \) be an \( \alpha \)-homeomorphism. That is \( f \) and \( f^{-1} \) are \( \alpha \)-continuous mappings. Since every IFCS in \( X \) is an \( \alpha \)-closed in \( Y \), \( f \) is an \( \alpha \)-closed mapping.

References